# The magnetoresistance and Hall coefficient of dilute Al-Si alloys



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# The magnetoresistance and Hall coefficient of dilute Al–Si alloys

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**Abstract.** The transverse magnetoresistance $\Delta\rho/\rho_0$ and the Hall coefficient $R_H$ of aluminium containing Si impurities has been measured at 4.2 K as a function of solute concentration. The simultaneous low-field magnetoresistance and Hall coefficient measurements permitted us to determine the anisotropy of the electronic scattering in terms of the mean free path ratios $l_{++}/l_-$ and $l_{--}/l_-$.

## 1. Introduction

For the last 10 years the galvanomagnetic measurements in aluminium alloys have been the subject of intensive study in order to understand the anisotropic scattering of conduction electrons by impurities (Boening *et al* 1975, Kesternich *et al* 1976, Sato *et al* 1978, McAlister *et al* 1979, Papastaikoudis *et al* 1979, Barnard and Abdel-Rahiem 1980). This anisotropy in the mean free path depends upon the local symmetry of the wavefunction on the Fermi surface and the symmetry of the impurity scattering centres. From Hall coefficient $R_H^0$ measurements in the low-field limit ($\omega_c\tau \ll 1$, where $\omega_c$ is the cyclotron frequency), one can obtain the most direct information about the transport relaxation time $\tau_k$ or mean free path $l_k$. Similarly, the low-field magnetoresistance was found to be sensitively dependent upon the details of such anisotropic scattering.

There appears to have been three previous studies of the low-field Hall coefficient $R_H^0$ in **Al**–Si alloys. Koester and Frei (1953) measured the room-temperature Hall coefficient in an alloy containing 0.52 at.% Si as a function of annealing at temperatures between 0 and 600 °C obtaining values of $R_H$ in the range from $-3.0 \times 10^{-11}$ to $-3.2 \times 10^{-11}$ m$^3$ C$^{-1}$. Matsuda and Sato (1966) determined the Hall coefficient for an alloy containing 0.59 at.% Si at 500 °C in order to avoid the influence of precipitations on $R_H$. Finally Brandley and Stringer (1973) measured the Hall coefficient in a series of dilute binary **Al**–Si alloys at 20 °C and they found a nearly linear relationship between $R_H$ and Si content.

In these three measurements the Hall coefficient includes phonon scattering and it is very difficult to draw conclusions about the anisotropic scattering of the Si atoms. For this reason, in order to exclude the influence of the phonon on the scattering mechanism, we have repeated the Hall measurements at 4.2 K.

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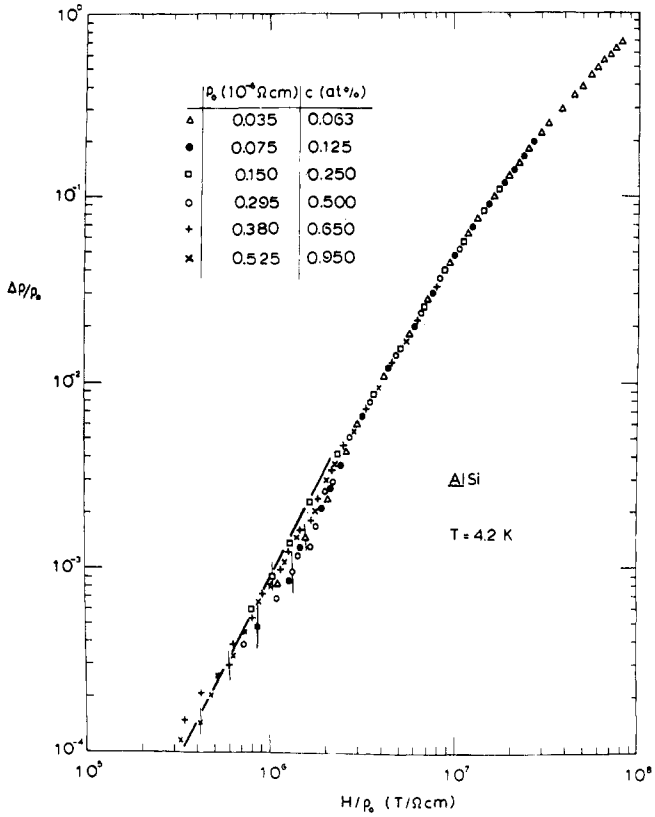


**Figure 1.** Kohler plot of the relative magnetoresistance $\Delta\rho/\rho_0$ of Al–Si at $T = 4.2$ K as a function of the 'effective' magnetic field $H/\rho_0$, where $\rho_0$ is the residual resistivity at $H = 0$.

## 2. Experimental procedure

The dilute alloys were produced from high-purity (99.9999%) aluminium (VAW, Bonn, Federal Republic of Germany) and silicon (purity, 99.999%) by high-frequency levitation melting and then rolled to foils of about $80 \pm 10$ $\mu$m thickness. These foils have been shaped with a special punching tool to a form which consists of a main area (12 mm × 2 mm) with two central extensions for Hall contacts and two other extensions for the magnetoresistance contacts. All alloys were annealed for 24 h at 550 °C in a vacuum better than $10^{-5}$ mbar. The alloy with 0.900 at.% Si was quenched very rapidly from 550 °C into a bath of methyl alcohol kept at −90 °C.

The impurity concentration of these alloys were determined by chemical analysis to an accuracy of ±1% for Si. The magnetoresistance and Hall voltage measurements were carried out in a conventional stainless steel helium cryostat, which contained a superconducting solenoid. All measurements were performed at 4.2 K in order to eliminate the effects of electron–phonon scattering. Further experimental details have been described previously (Papastaikoudis *et al* 1979).

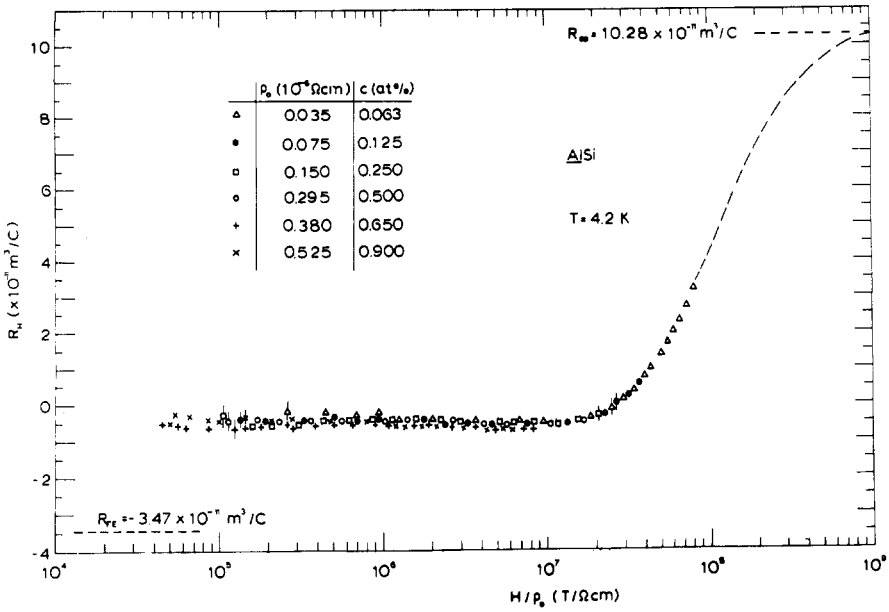


**Figure 2.** Kohler plot of the Hall coefficient $R_H$ of Al–Si at $T = 4.2$ K as a function of the 'effective' magnetic field $H/\rho_0$.

## 3. Results

The residual resistivity $\rho_0$ (4.2 K) of the currently investigated Al–Si alloys is proportional to the solute concentration, indicating that the Si atoms are homogeneously distributed. The residual resistivity per atomic per cent is about $0.580 \times 10^{-6}$ Ω cm at.%$^{-1}$, which is in excellent agreement with the calculated value of $0.580 \times 10^{-6}$ Ω cm at.%$^{-1}$ (Fukai 1969), while it differs from the values of $0.650 \times 10^{-6}$ Ω cm at.%$^{-1}$ obtained by Kawata and Kino (1975), $0.720 \times 10^{-6}$ Ω cm at.%$^{-1}$ obtained by Fickett (1971) and $0.670 \times 10^{-6}$ Ω cm at.%$^{-1}$ obtained by Takamura (1965). The probable reasons for the differences between the present and the previous values are believed to be the existing impurities in silicon and the metallurgical preparation of the samples.

Figures 1 and 2 show the relative magnetoresistance $\Delta\rho/\rho_0$ and the Hall coefficient $R_H$ of the Al–Si alloys at 4.2 K. Both $\Delta\rho/\rho_0$ and $R_H$ are plotted in the Kohler representation, i.e. as a function of the effective magnetic field $H/\rho_0$. At low fields there is a quadratic relationship between $\Delta\rho/\rho_0$ and the effective field $H/\rho_0$ (full line in figure 1), and the Hall coefficient $R_H$ exhibits constant and negative values which lie between $-0.4 \times 10^{-11}$ and $-0.6 \times 10^{-11}$ m$^3$ C$^{-1}$. For comparison, figure 2 shows the isotropic free-electron value Hall coefficient $R_{FE} = -3.47 \times 10^{-11}$ m$^3$ C$^{-1}$ for the aluminium and also the theoretical value of the high-field region Hall coefficient $R = 10.28 \times 10^{-11}$ m$^3$ C$^{-1}$. For higher values of $H/\rho_0$ the measurements tend to the high-field limit.

## 4. Discussion

The Kohler rule is a fundamental scaling law for galvanomagnetic coefficients and is valid exactly for the magnetoresistivity $\Delta\rho/\rho_0$ and Hall coefficient $R_H$ only if the

concentration but not the type of scattering centres is varied. This scaling law is valid for non-magnetic point defects and for any magnitude of magnetic field, as long as quantisation of the electron orbits or breakdown phenomena do not occur. Also changes in the host metal band structure can be neglected. The difference between the free-electron value $R_{FE}$ and the measured low-field Hall coefficient $R_H^0$ of the **Al**–Si alloys can be explained in terms of the electron scattering at silicon impurities and in terms of the nature of the Fermi surface. The electron scattering produces an extremely pronounced anisotropy of the electron mean free path $l_k$. The electron mean free path $l_k$ depends both on the electronic band structure of the aluminium and on the scattering potential of the impurities. Clearly, the low-field Hall coefficient $R_H^0$ is best suited to experimental studies of the anisotropy of the mean free path because $R_H^0$ is isotropic in cubic metals and described by the simple Tsuji (1958) formula. In the present discussion it is assumed that the possible changes in the shape of the Fermi surface of aluminium due to the insertion of Si impurities in the concentration range are too small to affect the low-field $R_H^0$ significantly.

The Fermi surface of aluminium is subdivided into three regions (Kesternich *et al* 1976). These are (i) a free-electron-like portion in the second zone which covers most of the Fermi surface, (ii) hole cylinders, also in the second zone, of very high positive curvature and (iii) electron-like cylinders, in the third zone, of very high negative curvature. These regions are assumed to be characterised by mean free paths $l_-$, $l_{++}$ and $l_{--}$, respectively. The Fermi surface regions with a high mean curvature have an extraordinary strong influence on the low-field Hall coefficient $R_H^0$ in comparison with the spherical part. According to this three-group model of conduction electrons of aluminium, Kesternich (1976), using the Jones–Zener series expansion method of the linearised Boltzmann equation, has obtained the following simple formulae for the relative magnetoresistivity $\Delta\rho/\rho_0$ and the low-field Hall coefficient $R_H^0$

$$\Delta\rho/\rho_0 = 50(l_{--}^3 + l_{++}^3/l_-^3)(R_{FE}H/\rho_0)^2$$
$$R_H^0 = R_{FE}[1 + 1.875(l_{--}^2 - l_{++}^2/l_-^2)]$$

where $R_{FE}$ is the isotropic free-electron value. Using these two equations the anisotropy of the mean free path of Si atoms in the ratios of $l_{--}/l_-$ and $l_{++}/l_-$ can be obtained from simultaneous measurements of $\Delta\rho/\rho_0$ and $R_H^0$. Using $\Delta\rho/\rho_0 = 2.5 \times 10^{-3}$, $R_H^0 = -0.50 \times 10^{-11}\ \mathrm{m^3\,C^{-1}}$ and $H/\rho_0 = 2 \times 10^6\ \mathrm{T\,\Omega cm^{-1}}$ we find that $l_{--}^{Si}/l_- = 0.63$ and $l_{++}^{Si}/l_- = 0.92$ or $l_{++}^{Si} > l_{--}^{Si}$. This ratio is of course only a rough estimate, but clearly the analysis requires the third-zone electrons to have a mean free path considerably smaller than that of the second-zone electrons. This means that the main scattering on Si impurities arises from electrons on the electron-like regions in the third zone. This anisotropy of $l_k$ originates physically from the different symmetry character of the involved electron wavefunctions $\psi_k(\boldsymbol{r})$ and the scattering potential of the defect. Possible differences of $\psi_k(\boldsymbol{r})$ could be especially large in Al since the important cylindrical regions of the Fermi surface lie on both sides of intersecting Brillouin zone boundaries. On the free-electron-like regions of the Fermi surface the wavefunction $\psi_-(\boldsymbol{r})$ is a plane wave. At the edges, the mixing of two plane waves results partly in standing waves, the amplitude of which is greatest at interstitial positions for the edges in the second zone ($|\psi_{++}(\boldsymbol{r})|^2$) and at lattice positions for the edges in the third zone ($|\psi_{--}(\boldsymbol{r})|^2$). The local charge densities obtained from four OPW calculations by Boening *et al* (1975) confirm this general characterisation of $\psi_-$, $\psi_{++}$ and $\psi_{---}$. Therefore the electrons on the hole-like edges in the second zone will be preferentially scattered at defects near interstitial positions and the electron-like edges in the third zone at defects near lattice positions.

The electrons on the free-electron-like Fermi surface regions will be scattered with similar probabilities on both types of defect. The Si impurities in the solid solution state take substitutional positions in the aluminium lattice and therefore according to the above considerations they will scatter strongly the conduction electrons in third zone, while relatively little scattering will occur from electrons (or rather holes) in the corners of the second zone. Thus it is expected that $l_{++} > l_{--}$ for the Si impurities, which explains the observed anisotropy.

## 5. Conclusion

The present measurements of the transverse magnetoresistivity and the low-field Hall coefficient for a series of Al–Si alloys at 4.2 K have shown that $l_{++} > l_{---}$. This result can be interpreted in terms of anisotropy of electron scattering from the Si impurities.

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